Local mass transfer from a circular cylinder in a uniform shear flow

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(Received 17 November 1989 and in final form 21 February 1990)

Abstract—A naphthalene sublimation technique is used to investigate the rate of mass transfer from a circular cylinder in a turbulent uniform shear flow. The rate of shear and the mean flow Reynolds number are varied to study their effects on the mass transfer. It is found that the distribution of the local mass transfer rate on the circular cylinder is characterized by a shear parameter \( K_d \) defined as \( S^*d/U_c \), where \( S^* \) is the shear rate, \( d \) the cylinder diameter and \( U_c \) the centreline mean velocity. The angular position at which the Sherwood number is minimum is approximately proportional to \( K_d \) on both the upper and the lower surfaces of the cylinder. The overall mass transfer rate is largely independent of \( K_d \) for the range of the present measurements \( 0 < K_d < 0.132 \), but it is strongly dependent on the centreline mean Reynolds number.

INTRODUCTION

A large number of experiments have been carried out on the heat and mass transfer from a circular cylinder. However, most of those have been concerned with the case where the cylinder is exposed to a uniform cross flow, and the heat and mass transfer characteristics from a circular cylinder subjected to a cross flow of uniform shear has not yet been dealt with.

Sogin and Subramanian [1] measured the local mass transfer from a circular cylinder in a uniform cross flow, and the results were compared with the approximate boundary layer calculation in the laminar region. They found that the calculations were reliable in the region between the stagnation point and the point of separation. Kestin and Wood [2] investigated the influence of free stream turbulence on the mass transfer from a circular cylinder using paradichlorobenzene (p-C6H4Cl2). In recent years, experimental investigations on various convection-dominated mass transfer have been conducted by using a naphthalene sublimation technique. Goldstein and Karni [3] examined the effect of the wall boundary layer on the local mass transfer from a circular cylinder. Mayle and Marziale [4] studied spanwise variations of mass transfer from a portion of a cylinder due to turbulence non-uniformity in the mainstream. Van Dresar and Mayle [5] investigated the convection rate at the base of the cylinder due to a horseshoe vortex, and reported a large spanwise variation in the mass transfer within the distance of one incident boundary layer height from the base surface. Goldstein et al. [6] carried out an experiment to measure the mass transfer from a square cylinder in a cross flow and from the surface on which the cylinder is mounted. They asserted that the variation of either the Reynolds number or the boundary layer thickness did not change the location of peaks created by the horseshoe vortex system, and only the Reynolds number was found to affect the magnitude of the mass transfer rate.

The same naphthalene sublimation technique is employed in the present study to measure the local rate of mass transfer from a circular cylinder immersed in a uniform shear flow. The centreline mean Reynolds number and the mean rate of shear are systematically varied to obtain the shear parameter \( K_d \) which is defined by \( S^*d/U_c \), in a range from 0.04 to 0.132. The influences of the Reynolds number and of the shear parameter on the mass transfer rate are analysed by processing our experimental data.

For the present study, an experimental facility was needed to generate a homogeneous uniform shear flow. An experimental realization of such homogeneous shear flow was first achieved by Rose [7]. He used a plane grid which consisted of parallel rods of uniform diameter with non-uniform spacing. The purpose of this experiment was to investigate the effects of shear on the development of the structure of turbulence. Since then, a number of similar experiments have been performed, with slightly different forms of the uniform shear generator [8–11]. For the present experiment, a new shear flow generator has been developed. Construction of the shear generator and the development of the velocity field will be described in the following sections.

BASIC EQUATIONS FOR MEASUREMENT OF MASS TRANSFER RATE

According to Fick's fundamental equation of diffusion, the mass flux at the naphthalene surface may be given by

\[
m = h_m (\rho_{e,w} - \rho_s) \tag{1}
\]
where $h_m$ is the coefficient of mass transfer and $\rho_{n,\text{w}}$ and $\rho_n$ denote the naphthalene vapour concentrations at the wall and at the outer free stream, respectively. The free stream vapour concentration outside the boundary layer is evidently zero. The local sublimation mass transfer from the naphthalene surface per unit area and time can be obtained by

$$m = \rho_n \cdot \Delta t / \Delta t$$

Combining equations (1) and (2), the mass transfer coefficient can be determined from

$$h_m = h_m \cdot d / D_i$$

It is assumed that the naphthalene vapour is a perfect gas so that $\rho_{n,\text{w}} = P_v / (R \cdot T)$. Here, the naphthalene vapour pressure $P_v$ is calculated from the partial pressure at saturation, which can be evaluated from a semi-empirical formula as a function of temperature [6, 12]

$$P_v(T) = \exp (31.49 - 8673/T)$$

where $D_i$ is the diffusion coefficient of the naphthalene into air and $d$ the diameter of the circular cylinder.

**EXPERIMENTAL APPARATUS AND METHODS**

*Wind tunnel, shear flow generator and velocity measurements*

Experiments have been conducted in an open-circuit blower-type wind tunnel. The wind tunnel was constructed based on the design guide of Mehta and Bradshaw [13]. The air speed at the square test section (450 mm x 450 mm) was varied from 2 to 33 m s$^{-1}$, and the level of free stream turbulence intensity was less than 0.5% at the exit of contraction. The settling chamber of the wind tunnel contains three sheets of screens (open area ratio, $\beta > 0.6$) and a honeycomb of hexagonal cells. The contraction area ratio was 8.35 : 1, and the shape of contraction was determined by the method of Morel [14].

A shear flow generator was installed at the exit of the contraction nozzle (Fig. 1). In order to produce the desired velocity profile, the flowing duct of the shear generator was divided into ten equally-spaced narrow channels. At the entrance of each channel, a butterfly damper was installed to control the flow rate through each channel. A small streamlined hill was located behind the damper to reduce the recirculatory wake flow. In addition, a circular rod of 8 mm diameter was installed at the exit of each narrow channel to reduce the length scale of turbulence and to improve the two-dimensionality of the uniform shear field.
The sidewalls were adjusted divergently in order to compensate for the static pressure loss. When the desired velocity profile was obtained by adjusting the dampers, the uniformity of the shear rate across the test section was checked by using a vertical rake of total pressure probes which were fabricated by 2.12 mm diameter stainless steel tubes. In order to minimize the flow disturbance due to the presence of the probes, the vertical distance between the probes was set at 2 cm, and the distance between the probe hole and the back stem of the rake was 50 tube diameters, as was suggested by Pope and Harper [15].

The measurement of the background turbulent flow field in the test section was carried out by using a two-channel hot-wire anemometer of constant temperature type. The x-wire probe (TSI, model 1241) was made of platinum wire of diameter 6 µm and length 1.25 mm. The wire overheat ratio was 1.5 and the frequency response was set at 15 kHz. The effective inclination of the x-wire to the free stream direction was obtained from the yaw test. Each analogue voltage signal from the two anemometers was digitally sampled by means of an A/D converter and a signal analyser, Data 6000 (Data Precision Model 611), with a data sampling rate of 10,000 s⁻¹.

Casting of naphthalene cylinder and measurement procedure
In order to measure the local rate of mass transfer from the cylinder surface in an approaching shear flow, a thin naphthalene layer was coated on a smooth cylinder surface. The cylinder made of stainless steel consisted of three separable parts, and a recess of about 2 mm deep was machined into the outer surface of the centre part of the test cylinder for the naphthalene casting (Fig. 2). The centre section of the cylinder was coaxially inserted inside the mould, and the molten naphthalene was poured outward into the bottom of the recessed space through two injection holes. In order to separate the mould from the naphthalene-cast cylinder without damaging its surface, the difference of the thermal expansion rates between the aluminium mould and the naphthalene casting was utilized. After a considerable number of trials, it was found that sudden heating of the mould up to about 76°C allows the expansion of the mould only, without transferring heat to the naphthalene. During the casting process, two thermocouples were embedded in the mould and on the surface of the naphthalene cylinder.

A precision roundness measuring instrument (RONOCOM 10A-1) was used to measure the local depth of naphthalene sublimation after exposure to the shear flow. The depth of sublimation was amplified 20,000 times for accurate measurement. Such fine resolution was sufficient to measure the actual change in the naphthalene thickness which varied from about $8.5 \times 10^{-3}$ to $5 \times 10^{-4}$ mm min⁻¹. The difference between the initial circumferential shape and the final one represents the amount of the sublimated naphthalene. The naphthalene sublimation is very sensitive to the environmental temperature. Thus, all the measurement procedures were performed in an air-conditioned laboratory in which the temperature was controlled within 0.3–0.5 °C variations.

The experimental uncertainty in the present measurement of the local mass transfer $h_a$ given by equation (3) may be assessed according to the uncertainty estimation method described in Holman [16] as follows. The uncertainty in $h_a$ arises from two kinds of errors. One stems from the estimations of the material
properties and the other is caused by the measurement error. Since the ambient temperature varied during the measurement within 0.5°C, the estimation of the material property, particularly of $\mu_{\infty}$, has an uncertainty of about 5%. This amounts to about 5.2% uncertainty in $h_m$ measurement. The measurement error is mainly due to the resolution limitation of the roundness measuring instrument, which is about 10% of the total sublimation depth for an exposure time of about 100 min. This yields the uncertainty in $h_m$ of about 9.8%. Therefore, the experimental uncertainty in the final evaluation of $h_m$ is approximately 11%, which is comparable to other experiments [17].

RESULTS AND DISCUSSION

The velocity field measurements

Figure 3 exhibits the two-dimensionality of the streamwise mean velocity distribution at $x/H = 7.5$ downstream from the shear generator for various heights $y$. Far from the boundary layers on the side-walls, the spanwise variation of the mean velocity was small which is comparable to that for other homogeneous flows [8, 9]. Representative vertical profiles of the mean velocity are shown in Fig. 4(a), where $U_c$ is the centreline mean velocity and $S$ the velocity gradient of the shear flow. The boundary layer on the upper plate is thicker than that on the lower one. Aside from these boundary layers, the mean velocity has a nearly uniform shear in the region $0.05 < y/H < 0.88$.

It has been found that, under the same setting of the shear generator resistance, increasing the centre-line mean velocity results in the augmentation of the rate of mean shear. However, the mean shear parameter $K_H$, defined by $K_H = S \cdot H/U_c$, remained constant for these cases. Here, $S$ is the shear rate, $H$ the test section height and $U_c$ the centreline mean velocity.
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The transverse variations of the r.m.s. velocities and the Reynolds shear stress at $Re_\tau = 48000$ are shown in Fig. 5, which reveals that the r.m.s. velocities have better homogeneity than the Reynolds shear stress. The homogeneity of $u'$ and $v'$ in our experiments is comparable to others [9, 10], however, no previous data of the Reynolds shear stress are available for comparison.

The developments of the r.m.s. velocities and the Reynolds shear stress along the centreline of the wind tunnel are shown in Figs. 6-8 for the lowest and highest Reynolds numbers used in the present mass transfer experiments. For the range of the present Reynolds numbers, they are almost independent of $Re_\tau$, but strongly dependent on $K_d$ during the initial development time $\tau = (x/U_\tau) \cdot (dU/dy)$. It may be noted that the downstream variations of all turbulence quantities for lower $K_d$ asymptotically approach to those of their respective quantity for higher $K_d$. The streamwise integral length scale $L_{11}$ was obtained by integrating the corresponding auto-correlation coefficient to its first zero crossing and using Taylor’s hypothesis (Fig. 9). Due to a relatively large uncertainty in the length scale measurement by this method as in refs. [8, 10], a strict linearity cannot be observed. However, it shows that the length scale grows monotonically with the downstream distance. All turbulence quantities of interest at the position of the mass transfer measurement $x/H = 7.0$ are summarized in Table 1. Such values for other $K_d$ and $Re_\tau$ within the ranges of the present measurements, may be approximately found by interpolations between these data.
Table I. Background turbulence data for the mass transfer experiments at $v/H = 7.0$ and $v/H = 0.5$

<table>
<thead>
<tr>
<th>$Re_c$</th>
<th>$K_d$</th>
<th>$n$</th>
<th>$-nu/\bar{U}_c^2$</th>
<th>$k/\bar{U}_c^2$</th>
<th>$L_{11}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24000</td>
<td>0.132</td>
<td>2.625</td>
<td>$1.699 \times 10^{-4}$</td>
<td>$1.713 \times 10^{-4}$</td>
<td>26.7</td>
</tr>
<tr>
<td>0.085</td>
<td>5.578</td>
<td>$2.043 \times 10^{-4}$</td>
<td>$6.443 \times 10^{-4}$</td>
<td>32.3</td>
<td></td>
</tr>
<tr>
<td>0.132</td>
<td>8.663</td>
<td>$4.817 \times 10^{-4}$</td>
<td>$1.438 \times 10^{-4}$</td>
<td>37.1</td>
<td></td>
</tr>
<tr>
<td>48000</td>
<td>0.132</td>
<td>2.625</td>
<td>$2.524 \times 10^{-4}$</td>
<td>$2.091 \times 10^{-4}$</td>
<td>30.8</td>
</tr>
<tr>
<td>0.085</td>
<td>5.578</td>
<td>$2.361 \times 10^{-4}$</td>
<td>$6.896 \times 10^{-4}$</td>
<td>40.4</td>
<td></td>
</tr>
<tr>
<td>0.132</td>
<td>8.663</td>
<td>$5.637 \times 10^{-4}$</td>
<td>$1.703 \times 10^{-1}$</td>
<td>47.7</td>
<td></td>
</tr>
</tbody>
</table>

The mass transfer measurements

The heat transfer coefficient around a circular cylinder at low Reynolds number in a cross flow has a minimum value at the point of separation [12, 18]. As $\theta$ increases from zero at the stagnation point, the static pressure on the cylinder surface decreases and the boundary layer thickness gradually increases. These cause a gradual decrease in heat transfer and the minimum point occurs near the point of separation. Then, there is a subsequent increase in the heat transfer coefficient on the rear side of the circular cylinder. However, there are two local minima at higher Reynolds numbers [12, 18]. The first one occurs at the point of transition from the laminar to the turbulent boundary layer and the second at a point from which the turbulent boundary layer is separated. Because of the complicated nature of the flow separation process, it is difficult to calculate the average heat transfer coefficient in a cross flow. A comparison of an approximate boundary layer solution before separation with the present experimental results is shown in Fig. 10. The dotted line corresponds to Frossling's theoretical solution for the laminar boundary layer on a circular cylinder. Here, the heat transfer rates were converted into mass transfer rates by using an appropriate analogy between heat and mass transfers. Direct comparison of the theory for uniform flow with the experimental result in uniform shear flow is not appropriate; however, it can be stated that the local rate of mass transfer around the upper and lower circular cylinder in the uniform turbulent shear flow is higher than that in the uniform laminar flow.

If the approaching flow is uniform ($K_d = 0$) for $Re_c < 10^5$, the local mass transfer coefficient has a minimum value near $80^\circ$ from the front stagnation point [6, 18]. However, in the mean shear flow, the angle corresponding to the minimum Sherwood number is near $85^\circ$ on the upper surface and near $75^\circ$ on the lower surface of the circular cylinder, as shown in Fig. 10. The decreasing rate of $Sh$ along the angle from the front stagnation point on the upper surface is found to be slower than that on the lower one.

The variations of the local mass transfer at a fixed shear parameter, $K_d = 0.132$ for different Reynolds numbers on the upper and lower surfaces are shown in Figs. 11(a) and (b), respectively. It can be seen that the qualitative variation of the Sherwood number with $\theta$ is different for different Reynolds numbers.
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$\theta$ has a rather consistent shape with a fixed shear parameter even though the Reynolds number and the shear rate vary.

Although the local Sherwood number is a function of $Re_c$, the minimum of the local $Sh$ occurs at nearly the same location for different shear rates and Reynolds numbers. It is also noted that the local $Sh$ after the separation point first reaches its local maximum and minimum and then gradually increases with $\theta$ on both the upper and lower surfaces.

The variation of the local $Sh$ with different values of $K_d$ at a fixed $Re_c$ on the upper and lower surfaces are shown in Figs. 12(a) and (b), respectively. Comparing this variation with that in Figs. 11(a) and (b), it is concluded that the effect of $K_d$ on the Sherwood number is much weaker than that of $Re_c$. Before the separation point, the local Sherwood number is slightly higher for higher $K_d$ on the upper surface. (However, there is no such similar relation between $Sh$ and $K_d$ on the lower surface.) Recalling that the increase of the free stream turbulence intensity results in the augmentation of the local heat and mass transfers over a circular cylinder in a uniform cross flow as observed by Kestin and Wood [2] and Lowery and Vachon [19], the dependency of $Sh$ on $K_d$ in Figs. 12(a) and (b) seems to be more attributed to the difference in the intensity of free stream turbulence (see Figs. 6 and 7 and Table 1). Figure 13 shows the positions of minimum $Sh$ as a function of $K_d$. Increase of the shear parameter causes a gradual delay of the minimum position of $Sh$ in the upper part, while it pulls up the minimum position in the lower part.

In order to investigate the effect of Reynolds number around a circular cylinder on the average Sherwood number $\overline{Sh}$, a circumferential average Sherwood number is determined from the local measurements by

$$\overline{Sh} = \frac{1}{360} \sum_{\theta} Sh \cdot \Delta\theta,$$

where the increment in degrees ($\Delta\theta$) is an appropriate division of the measured angle in the circumferential direction. The results are shown in Fig. 14, which reveals that $Sh$ depends strongly on $Re_c$ but it does not show any consistent dependency on $K_d$. If the well-known power law $Sh = a \cdot Re_c^m$ is applied to these data, the index $m$ turns out to be about 0.6 which is a little higher than that of Zukauskas [18] for the mean heat transfer rate over the circular cylinder in a uniform cross flow.
CONCLUDING REMARKS

A naphthalene sublimation technique was employed to measure the local mass transfer rate around a circular cylinder in cross flow of uniform mean shear. The experiments have been performed in the ranges of \(0 \leq K_d \leq 1.238\) and \(24000 < Re_\phi < 48000\), where \(Re_\phi\) is the centreline mean Reynolds number.

In this range of \(Re_\phi\), the angular positions of minimum Sherwood number on the upper cylindrical surface and on the lower one do not exhibit much dependence on \(Re_\phi\), but they vary almost linearly with the shear parameter \(K_d\); an increase in \(K_d\) delays the upper position while it pulls up the lower one.

The magnitude of the local Sherwood number, however, strongly depends on \(Re_\phi\). The weak dependency of the local Sherwood number on \(K_d\) observed in the present measurements may be due to the difference in intensity of free stream turbulence for different \(K_d\).

The average Sherwood number around the circular cylinder also depends strongly on \(Re_\phi\), but the dependence on \(K_d\) is minor. If the power law \(Sh \sim Re_\phi^{m}\) is assumed, the index \(m\) turns out to be about 0.6, which is a little higher than that of the power law correlation of the heat transfer around the circular cylinder in a uniform cross flow.

Acknowledgements—This research was performed under the auspices of Korea Science and Engineering Foundations. The authors would like to acknowledge the technical assistance of Soong Ki Kim and Kang Hyun Lee.

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TRANSFERT LOCAL DE MASSE A PARTIR D'UN CYLINDRE DANS UN ECOULEMENT

Résumé—Une technique de sublimation de naphthaléne est utilisée pour étudier le flux de masse convecté à partir d'un cylindre circulaire dans un écoulement turbulent de cisaillement. Le taux de cisaillement et le nombre de Reynolds de l'écoulement moyen sont variés pour étudier leurs effets sur le transfert de masse. On trouve que la distribution du coefficient local de transfert autour du cylindre est caractérisée par un paramètre de cisaillement \(K_d\) défini par \(S' \cdot dU_\phi\), où \(S'\) est le taux de cisaillement, \(dU_\phi\) diamètre du cylindre et \(U_\phi\) la vitesse moyenne au plan central. La position angulaire pour laquelle le nombre de Sherwood est minimal est approximativement proportionnelle à \(K_d\) et varie sur les surfaces supérieure et inférieure du cylindre. Le coefficient global de transfert de masse est largement indépendant de \(K_d\) dans le domaine des mesures effectuées \(0 < K_d < 0,132\), mais il est fortement dépendant du nombre de Reynolds sur le plan central.
Zusammenfassung—Mit Hilfe der Naphtalinsublimationstechnik wird der Stoffübertragung an einem Kreiszylinder in einer gleichförmigen turbulenten Scherströmung untersucht. Die Schubspannung und die mittlere Reynolds-Zahl der Strömung werden variiert, um den Einfluß auf den Stoffübertragung zu untersuchen. Dabei zeigt sich, daß die örtliche Verteilung des Stoffübertragungs am Kreiszylinder mit einem Parameter für die Schubspannung $K_d$ beschrieben werden kann. Dieser ist definiert als $S \cdot d / U_m$, wobei $S$ die Schergeschwindigkeit, $d$ der Zylinderdurchmesser und $U_m$ die mittlere Geschwindigkeit in der Strömungslinie ist. Die Stelle am Rohrumbau, an der die Sherwood-Zahl am kleinsten ist, verhält sich näherungsweise proportional zu $K_d$. Dies gilt sowohl für die obere wie auch die untere Oberfläche des Zylinders. Der Gesamtstoffübertragung ist weitgehend unabhängig von $K_d$ (gültig für $0 < K_d < 0.132$ bei den vorgestellten Messungen), er hängt jedoch stark von der mittleren Reynolds-Zahl im Kern der Strömung ab.