Vortex simulation of leading-edge separation bubble with local forcing

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Abstract

A numerical study is made of the flow control of leading-edge separation bubbles in a two-dimensional semi-infinite blunt plate, which is aligned parallel to the main stream. A version of the discrete-vortex method is utilized. A time-dependent point source of small magnitude is introduced near the separation edge. The effect of local source forcing on the evolution of a leading-edge separation bubble is scrutinized by varying the strength and the forcing frequency of the source. It is found that the reattachment length attains a single minimum at lower forcing levels, while it shows double minima at moderately high forcing levels. Based on the numerical results, the mechanism of decreasing reattachment length is pursued. These findings are qualitatively consistent with the existing experimental results for a blunt circular cylinder.

1. Introduction

Flow about a blunt body constitutes a basic problem. A typical flow pattern at high Reynolds numbers indicates the formation of a separation bubble near the sharp corner as well as the emergence of reattached flow further downstream. A paradigmatic flow configuration is illustrated in Fig. 1. This warrants in-depth study from a theoretical viewpoint. Also, the flow behavior is of interest for a multitude of practical technological applications. The presence of a separated flow, together with a reattaching flow, gives rise to unsteadiness, pressure fluctuations, structural vibrations, and noise, to cite a few effects. It is therefore an essential task to gain a proper understanding of the phenomenon and seek possible ways to reduce the above-stated adverse impacts.

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A literature survey reveals that there have been attempts to control or lessen the unfavorable behavior associated with the separation bubble. The deformation of the sharp separation edge of the blunt body was proposed by Gai and Sharma (1987). Use of sound waves to influence the reattachment process was examined by several researchers (Bhattacharjee et al. 1986; Roos and Kegelman 1986; Cooper et al. 1986; Zaman et al. 1987; Nishioka et al. 1990). In particular, the introduction of a local forcing in the vicinity of the separation edge has been contemplated (Sigurdson and Roshko 1988; Kiya et al. 1993). These experimental efforts utilized a small-amplitude localized jet flow very close to the square-cut leading-edge of the blunt body. The jet flow contained a well-defined single-frequency pulsation. The frequency and the amplitude of this pulsating jet varied in an experiment for a blunt-based circular cylinder (Kiya et al. 1993). It was clearly demonstrated that, by means of this small localized perturbation near the separation edge, the overall characteristics of the separation bubble were altered substantially.

The aforesaid method of controlling the separation bubble presents promising ground for further scrutiny. The crux is that some of the unfavorable characteristics of a separation bubble may be reduced by the introduction of a small, localized perturbation near the vicinity of the separation edge. The success of prior experimental studies indicates the need to perform detailed numerical studies to look into the feasibility of separation control. In view of the relative ease of numerical simulations, the present work reports the results of computational studies of a canonical flow configuration. The well-documented discrete vortex method will be used to secure descriptions of the separated flow at high Reynolds numbers (Kiya et al. 1982; Sarpkaya 1988). In particular, the discrete vortex method has been shown to be a powerful tool to describe the separated flow at high Reynolds numbers. The core of the method is that the leading-edge separation bubble is represented by a combination of two ingredients. One is the inviscid potential flow, and the other is the time-dependent behavior of elemental vortices, which are shed in the neighborhood of the
separation point. The effect of viscosity and the reduction of the strength of circulation of vortices after shedding are accounted for by invoking physically plausible models. The ability of the elemental vortices to adequately represent a continuous vortex sheet in the separation bubbles is demonstrated by Clements (1973) and Kiya et al. (1982).

In the present study, a time-dependent point source of small magnitude is introduced near the separation edge of a two-dimensional half-infinite flat plate with a square-cut leading edge. The principal rationale in selecting a source is that it be an idealized representation of the simplest possible perturbation to the flow. This allows an easy manipulation of numerical and mathematical formulations; yet it portrays the major features of control of a separation bubble by means of inserting a perturbation.

In this paper, we consider a flow past a two-dimensional flat plate with finite thickness, which is aligned parallel to a uniform approaching stream with a local forcing near the separation point (Fig. 1). By changing the forcing frequency and amplitude, the effects of local forcing on the evolution of leading-edge separation bubble are analyzed in terms of the reattachment length, distributions of mean velocity and turbulent quantities, profiles of the reverse-flow time fraction, and wall velocity. The motions of instantaneous reattachment in the space–time domain are demonstrated. In order to investigate the mechanism by which reattachment length of the separation bubble is decreased, the cross-correlation of pressure and velocity fluctuation and relevant convection velocities are also evaluated. As a qualitative verification of the present study, the experimental results by Kiya et al. (1993) are consulted, where the geometric configuration is the axisymmetric circular cylinder. The same geometry and the same mechanism of local forcing between experiment and simulation are certainly a desirable avenue; however, the present two-dimensional simulation qualitatively demonstrates characteristics similar to those experimental results.

2. Vortex simulation

2.1. Discrete vortex method

In order to describe the flow sketched in Fig. 1, the discrete vortex model is adopted. This has been successfully employed by several authors (e.g., Kiya et al. 1982; Chein and Chung 1988) to tackle a variety of separated flows. As reviewed in the literature (Sarpkaya 1988), the crux of this method is to represent the shear layer emanating from the separation points by an array of elemental vortices, which are termed the nascent vortices. The motion of the shear layers is described by the evolution of the arrays of vortices.

The leading-edge separation bubble of a blunt two-dimensional body is considered. The separation bubble is caused by flow separation from the sharp leading edge of a blunt two-dimensional body. The flat plate is of finite thickness \( H \) and semi-infinite length. The interactions between the two separation bubbles at the corners are assumed to be minimal. Thus, the symmetry condition is applied in this problem. The geometry of the body is given in Fig. 1.

For the problem formulation, the flat plate in the physical \( z \)-plane is mapped, using a Schwartz–Christoffel transformation, into the transformed \( \zeta \)-plane (Kiya et al. 1982):

\[
z = H/\pi \left[ (\zeta^2 - 1)^{1/2} - \text{arcosh} \zeta \right] + iH,
\]
where the corner of the body, \( z = iH \) and the origin of coordinate system are transformed into the points \( \zeta = 1 \) and \( \zeta = -1 \), respectively.

The complex velocity potential \( F(\zeta) \), representing the total flow in the transformed plane, is made up of the potential due to the approach flow and the potential induced by vortices:

\[
F(\zeta) = U_\infty (H/\pi)\zeta - \sum_{j=1}^{n} \frac{i\Gamma_j}{2\pi} \left\{ \log(\zeta - \zeta_j) - \log(\zeta - \zeta_j^*) \right\},
\]

where \( \Gamma_j \) and \( \zeta_j \) denote the circulation and the location of the \( j \)th element vortex, and \( \zeta_j^* \) is its complex conjugate.

Since sources and vortices maintain the same strength under conformal transformation, the velocity at a point in the physical \( z \)-plane is obtained by transforming the positions of all the vortices into the transformed \( \zeta \)-plane, calculating the velocity at the transform of the required point, and then returning this velocity back into the physical \( z \)-plane.

The velocity in the physical \( z \)-plane is therefore expressed as

\[
W(z_k) = \frac{U_\infty H}{\pi} + \sum_{j=1, j \neq k}^{n} \frac{i\Gamma_j}{2\pi} \frac{1}{\zeta_k - \zeta_j} - \sum_{j=1}^{n} \frac{i\Gamma_j}{2\pi} \frac{1}{\zeta_k - \zeta_j^*} - \sum_{j=1}^{n} \frac{i\Gamma_j}{4\pi} \frac{1}{\zeta_k^2} - 1 \right\} \left( \frac{dz}{d\zeta} \right)^{-1},
\]

where the last term represents Routh's correction (Clements 1973; Sarpkaya 1988).

The key step of the present numerical model is the determination of the strengths and positions of the nascent vortices. In order to avoid the unbounded magnitude of the velocity at the separation point, which is the singular point of transformation, the Kutta condition should be enforced at the same point, very close to the separation edge in the transformed \( \zeta \)-plane,

\[
\left( \frac{dF}{d\zeta} \right)_{\zeta = 1} = 0.
\]

The rate of vorticity shedding at the separation point can be approximated by the following relation (Evans and Bloor 1977):

\[
\frac{d\Gamma}{dt} = \frac{1}{2} U_s^2,
\]

where \( \Gamma \) is the strength (or circulation) of the nascent vortex and \( U_s \) is the velocity at the outer edge of the separated shear layer with a small width \( \varepsilon \). This approximation is adopted in the present study under the assumption that the outer flow velocity is much larger in magnitude than the inner flow velocity (Kiya et al. 1982; Nagano et al. 1982; Chien and Chung 1988; Sarpkaya 1988). The position of the nascent vortex is assumed to be the middle point between the separation point and the outer edge of the shear layer — i.e., \( z = i(H + 0.5\varepsilon) \). In order to determine the values of \( \Gamma \) and \( \varepsilon \), Eqs. (4) and (5) are solved iteratively at each calculation step.

The convection of vortices is stimulated by employing the Adams–Bashforth scheme:

\[
z_k(t + \Delta t) = z_k(t) + 0.5(3W^*(t) - W^*(t - \Delta t))\Delta t,
\]
where $\Delta t$ is the nondimensional time step, which is normalized by the reference time $H/U_\infty$. The time step is set to be sufficiently small to enable detailed description of the prominent characteristics of the evolving flows ($\Delta t = 0.1$).

As pointed out by Sarpkaya (1988), the discrete-vortex method does not deal directly with the viscosity effects. Several models have been proposed to address this aspect (Evans and Bloor 1977; Nagano et al. 1982; Smith and Stansby 1989; Chang and Chern 1991), although a precise treatment of viscosity remains an unresolved problem. The current status of model capabilities in this regard should be borne in mind. In the present paper, the experimental findings of Nagano et al. (1982) are cited to obtain an estimate of vorticity decays. In view of these experimental measurements, the vorticity decay is modelled as

\[
\frac{\Gamma(t)}{\Gamma_n} =
\begin{align*}
0.99, & \quad 0 < t < 2.0 \\
1 - \exp\left(-60.0/t\right), & \quad 2.0 < t < 8.0 \\
0.60, & \quad t > 8.0
\end{align*}
\]

in which $\Gamma_n$ is the initial strength, $\Gamma(t)$ is the circulation at time $t$.

Since the velocity field induced by a point vortex becomes unbounded near the vortex, the cut-off vortex model, proposed by Chorin (1973), was adopted in the present study. The cut-off vortex method indicates that the velocity within the cut-off radius is set to be a constant value. The value of the cut-off radius was determined in a manner similar to that of the preceding investigation (Kiya et al. 1982). In the actual calculations, no computations were made in the region very close to the solid plate, within a distance of 0.05H. In this region, owing to the proximity of the image vortices, unreasonably high velocities are expected to occur. In order to bypass these difficulties, the aforementioned Chorin's cut-off model was employed. This is admittedly not a completely satisfactory remedy. However, in light of the lack of a better model, this approach is adopted in the present work.

### 2.2. Source forcing vortex method

As briefly mentioned in the introduction, the numerical feasibility of separation control is scrutinized in the present study by utilizing a small, localized perturbation in the vicinity of the separation edge. A time-dependent point source of small magnitude is introduced near the separation edge. This is mainly due to the relative ease of numerical and mathematical formulations. The principal rationale of selection of a source is that it is an idealized representation of the simplest possible perturbation to the flow. Since the point source element induces a velocity which decreases rapidly, being inversely proportional to the distance from the source, it affects directly the elemental vortices in the vicinity of the source. It is revealed, however, that these confined effects have a great influence on the whole flow field.

As pointed out in the previous formulation, the separation point is a singular point of transformation. For an optical remedy, the source position ($z_s$) should be moved slightly downstream to
avoid an unbounded velocity at separation. Toward this end, the concept of a core radius \((\sigma_s)\) is introduced, which is similar to the cut-off radius of Chorin (1973) – i.e., the velocity within the core radius is assumed to be constant. In order to determine the optimum values of \(z_s\) and \(\sigma_s\), a number of preliminary calculations were made by varying these values. It is desirable to locate a source as close to the separation point as possible. However, too small a distance results in a singular behavior of flow. Similarly, a small value of the core radius gives rise to the singularity. On the contrary, a large value of the core radius leads to the attenuation of the local forcing in magnitude. In the present study, these optimum values are set \(z_s/H = 0.015\), \(\sigma_s/H = 0.01\). Chorin and Bernard (1973) reported that the effect of cut-off radius on the roll-up process is minor, provided that the time step is kept reasonably small. We also observed that the relative effect of core radius was small, while the position of the source had a significant impact on the flow near separation.

The complex potential, representing a two-dimensional flow around a blunt plate with a source forcing near the separation point, is written as

\[
F_p = U_\infty (H/\pi)z + (2\pi)^{-1} Q \log (z - z_s), \tag{8a}
\]

\[
Q = Q_0 \sin (2\pi f t), \tag{8b}
\]

where \(U_\infty\) is the time-mean velocity at upstream infinity and \(Q\), the strength of the source located at \(z_s\) in the transformed \(\zeta\)-plane. In Eq. (8b), the forcing frequency of the source is represented as \(f\), which is normalized by the reference frequency \(U_\infty/H\).

3. Results and discussion

It is important to ascertain the reliability and accuracy of the present numerical procedures. This forms an integral part of the overall validation efforts. Toward this end, the discrete vortex simulation of the separation bubble over a two-dimensional blunt flat plate, which is the flow without a source forcing, has been repeated. Prior results of experiment and simulation are available for comparison with the present results (Kiya et al. 1982; Kiya and Sasaki 1983).

Among the various data representing the separation bubble, the reattachment length \((x_R)\) is frequently used as a representative quantity in a time-mean sense. In order to find the reattachment length, the reverse-flow time fraction \(I_\tau\) or the time-mean surface velocity \(U_w\) near the surface should be measured. Thus, these two quantities are selected to compare the present simulation with the existing experimental data (Kiya et al. 1982).

The calculated distributions of the reverse-flow time fraction \(I_\tau\) on the plate surface in the vicinity of the wall \((y/H = 0.005)\) are represented in Fig. 2a, together with the experimental results of Kiya et al. (1982). In their experiment, the reattachment length is \(x_R = 9.6\). The distributions of the time-mean surface velocity \(U_w\) at the same height are also shown in Fig. 2b with the experimental data. The time-mean reattachment position \((x_{Rt})\) can be defined as the point where the reverse-flow time fraction has the value of \(I_\tau = 0.5\) or \(U_w = 0\). In both figures, the position of \(I_\tau = 0.5\) coincides exactly with that of \(U_w = 0\). It is encouraging to see that the present vortex simulation gives generally reasonable agreement with the experimental data.

Now, the performance of the present local source forcing is inspected. These forcing levels \((A_0 = 0.5, 1.0, \text{ and } 1.5)\) are chosen arbitrarily, where the forcing level \(A_0\) represents the velocity...
induced by the source \((Q_0)\) at the core radius \(\sigma_s\), i.e., \(A_0 = 10Q_0U_x/\pi\). The range of forcing frequency is \(0 \leq fH/U_x \leq 0.4\). As a measure of effectiveness of the local forcing, the normalized reattachment length of the separation bubble \(x_R/x_{R0}\) is plotted in Fig. 3 against the nondimensional forcing frequency. Here, \(x_{R0}\) denotes the time-mean reattachment length of the unforced flow \((A_0 = 0)\). Many calculations of local forcing have been made by varying the forcing level and frequency. As is shown in Fig. 3, the effect of local forcing is substantial. At lower forcing levels \((A_0 = 0.5\) and \(1.0)\) the reattachment length has a single minimum approximately at \(fH/U_x = 0.17\), while it shows double minima at the forcing level \(A_0 = 1.5\). These particular forcing frequencies may be referred to as the most effective in the sense that the reduction of separation region is usually sought in practical applications. A global feature of Fig. 4 indicates that the forcing levels affect only the total size of reattachment; i.e., the reattachment length generally decreases with increasing forcing levels.
In order to elucidate the evolution of the large-scale spanwise vortical structure of the separation bubble, the trajectories of elemental vortices for the unforced flow \((A_0 = 0)\) are demonstrated in Fig. 4. As can be seen, several vortex clouds are formed in the separated shear layer, and they coalesce to form larger and larger vortex clouds which are subsequently swept downstream (as
denoted by the dotted line). The average moving speed of vortex clouds ($0.5U_x$) is nearly equal to the convection velocity of the large-scale vortices, which is consistent with the experimental result of Kiya et al. (1982).

The trajectories of elemental vortices at successive times for an arbitrarily selected forced flow ($A_0 = 1.0$, $fH/U_x = 0.1$) are represented in Fig. 5. As compared with those of the unforced flow in Fig. 4, large-scale vortices in the forced flow are more visible. In addition, the rolling-up behavior into large-scale vortices is seen to be vigorous in the separation bubble. It appears from the figure that the large forced structures cause large increases in entrainment close to the separation point. The increased entrainment results in time-averaged streamlines with a small radius of curvature near the separation point. These are closely associated with the shrinkage of the bubble. The shedding of large-scale vortices becomes periodic, and the dominant forcing frequency can be detected in the power spectrum. The flow pattern in the downstream reattachment seems to be less coherent than that within the separation bubble. This feature can be interpreted in terms of the fact that the present source forcing has a substantial effect on the flow near the separation point.

Fig. 6a represents the calculated distribution $I_t$ at $y/H = 0.005$ for the forced flow ($A_0 = 1.0$, $fH/U_x = 0.1$). It is shown that the reattachment position ($x_R/H = 6.8$) of the forced flow

Fig. 5. Trajectories of elemental vortices over the separation bubble for the forced flow ($A_0 = 1.0$, $fH/U_x = 0.1$).
Fig. 6. (a) Distributions of the reverse-flow time fraction factor $I_r$. (b) Distribution of the time-mean surface velocity $U_w$.

is much shorter than that of the unforced flow ($x_{R_0}/H = 8.4$). As an indicator of unsteadiness strength in the reattaching zone, the streamwise span for the case of intermittent reattachment is often used. The streamwise span is defined as the streamwise distance, where $I_r$ belongs to the region $0.1 < I_r < 0.9$. In Fig. 6a, the streamwise span of the unforced flow is about $4.5H$, while the span of the forced flow is $6.5H$. This means that the forced flow has stronger large-scale vortices than the unforced flow.

Distributions of the time mean surface velocity $U_w$ at $y/H = 0.005$ are shown in Fig. 6b for both the forced and the unforced flow. The position of zero surface velocity in the forced flow ($A_0 = 1.0$, $fH/U_\infty = 0.1$) is located upstream of the reattachment position of the unforced flow. The positions where $I_r = 0.5$ and $U_w = 0$ are seen to be exactly coincident. In the vicinity of the plate edge ($x/H < 1.0$), the surface velocity is shown to be positive for both cases. This fact shows the existence of a secondary separation bubble, which has been reported in an experiment (Kiya et al. 1982).

Distributions of the time-mean streamwise velocity are displayed in Fig. 7a. The difference between the cases of a forced flow ($A_0 = 1.0$, $fH/U_\infty = 0.1$) and an unforced flow is seen to be
negligible near the separation point, where vortices are just formed. However, the effect of forcing becomes more pronounced in the separation bubble. It is also seen that the effect of local forcing is significant in the separation bubble. The height of the separation bubble for the forced flow is slightly lower than that of the unforced flow, which is consistent with the experimental findings of Sigurdson and Roshko (1988). Fig. 7b shows the streamwise turbulent levels for the forced flow. The reverse flow intensity of the forced flow is much stronger than that of the unforced flow. Stable vortex pairing induced via forcing can increase the overall turbulent intensity level compared to the unforced flow.

The flow in the reattaching zone is unsteady, as it is dominated by the motions of the large-scale vortices (Eaton and Johnston 1982). The unsteady motion in the reattaching zone has been reported in Kiya and Sasaki (1985), where the unsteadiness was demonstrated by the motion of reverse-flow regions. They characterized a boundary between a reverse-flow region and a forward-flow region, i.e., \( u_0 = 0, \frac{\partial u_0}{\partial x} < 0 \) or \( u_0 = 0, \frac{\partial u_0}{\partial x} > 0 \), respectively. Here, the surface velocity \( u_0 \) was defined as the short-time average velocity very close to the surface. Fig. 8a shows
the unsteady motion of the unforced flow in the space–time domain. The instantaneous positions of $u_s = 0$, $\partial u_s / \partial x < 0$ and of $u_s = 0$, $\partial u_s / \partial x > 0$ are represented by symbols $\bigcirc$ and $\times$, respectively. In this figure, one or two reverse-flow regions in the reattaching zone are observed; in actuality, the number of reverse-flow regions depends upon time.

The unsteady motion in the reattaching zone is shown in Fig. 8b for a particular forcing case ($A_0 = 1.0$, $fH/U_\infty = 0.1$). In contrast to the unforced flow in Fig. 8a, in the forced flow the unsteady motion becomes more organized and vigorous. The span of reattachment of the forced flow is wider than that of the unforced flow. This feature suggests that the forced flow in the reattaching zone is strongly influenced by large-scale vortices. It is also found that the frequency of shedding of the large-scale vortices is nearly equal to the actual forcing frequency.

As pointed out previously, the reduced reattachment length is closely related to the increased growth rate of the shear layer and its curvature toward the wall—i.e., an increase in entrainment. It is evident that the growth rate depends on how well the shear layer is forced (Hasan 1992). In an attempt to understand the mechanism of the effect of forcing, the cross-correlation without time lagging between the pressure fluctuation at the reattachment point and the streamwise velocity fluctuations in the streamwise direction, $R_{pu} = \frac{\bar{p}'(x_{R}, t)}{\bar{u}'(x_{R}, t)}$ at $y/H = 0.005$ is shown in Fig. 9. The correlations were obtained for three different forcing frequencies ($A_0 = 1.0$, $fH/U_\infty = 0.06$, 0.12, and 0.18). The abscissa $x$ is normalized by the corresponding reattachment length $x_R$. In this figure, the cross-correlations show unstable changes at the separation edge and then display
regular oscillations further downstream, attaining maxima near the reattachment region. It is reasonable to assume that the spacing between the negative lobes (or peak to peak) represents a particular streamwise length scale in the separation bubble. Accordingly, this length scale can be regarded as the spatial distance between the large-scale vortical structures near the reattachment zone (Cherry et al. 1984). The distance seems to be decreased as the forcing frequency increases. This means that more vortex clusters are compressed within the separation bubble as the forcing frequency increases.

Fig. 10 shows the corresponding time-lagged correlations for the unforced flow, $R_{pu} = p'(x_R, t) u'(x, t + \tau)$, at $y/H = 0.005$. The time delay $\tau$ is positive for velocity delayed with respect to pressure. The convective motion near the reattachment zone is clearly displayed in the free-stream direction; i.e., the large-scale vortices move downstream. The convective velocity $U_c$ can be obtained by measuring the moving velocity of the maximum of correlation. The present results are in good agreement with the results of Cherry et al. (1984).
The locations of maximum positive peaks are plotted against the corresponding time delay $\tau_{\text{max}}$ in Fig. 11, where $\tau_{\text{max}}$ is the time delay to the peak positive correlation for $R_{pu}$. This provides a measure of convective velocity. In the reattachment region, the slope at the origin gives a measure of convection velocity. As stated earlier, the convection velocity in the unforced flow is approximately $0.5U_\infty$. As the forcing frequency increases, the convection velocity also increases. However, a closer inspection reveals that the slope of $fH/U_\infty = 0.06$ is lower than that of the unforced flow ($fH/U_\infty = 0$). This can be explained by the fact that the shear layer forced at $fH/U_\infty = 0.06$ suppressed the overall turbulence below the level of the unforced flow. This suppression at a special forcing frequency was also observed in the experimental results of Zaman and Hussain (1981) and Hasan (1992).

It is recalled that, for an axisymmetric blunt circular cylinder, Kiya et al. (1993) performed extensive experiments on flow control. A single-frequency sinusoidal disturbance was introduced along the square-cut leading edge through a thin slit at high Reynolds number. In their experiment, the forcing level was defined as the values of r.m.s. velocity fluctuation at a particular position near the separation edge, where the value of time-mean velocity is 90% of the maximum value. Their experimental results on the normalized reattachment length $x_R/x_{Ro}$ are plotted in Fig. 12 against the nondimensional forcing frequency for three forcing levels $(u'^2 + v'^2)^{0.5}/U_\infty = 0.005, 0.001, \text{ and}$
Fig. 11. Spatial separation ($\Delta x/H$) against the time delay for maximum correlation ($\tau_{\text{max}} U_\infty/H$).

Fig. 12. Normalized reattachment length against forcing frequency (Kiya et al. 1993).

0.02. It is noted that, even though the flow geometry is different, the two results are qualitatively consistent with each other. As shown in Fig. 3, a single minimum of the reattachment length is also attained for small forcing levels; however, it is surprising that two minima appear at appropriate forcing levels. It is also apparent that the reattachment length decreases with increasing forcing levels.

4. Conclusion

A version of the discrete vortex simulation was applied to calculate the feasibility control of the leading-edge separation bubble. A time-dependent point source near the separation edge was introduced into the simulation, which proved to be effective in dealing with the flow control. As demonstrated previously, a single minimum of the reattachment length is attained at lower forcing
levels, whereas the reattachment length shows double minima at a moderate forcing level. The unsteady motion in the reattachment zone for the case of forcing is found to become more organized and vigorous than that of the unforced flow. From the distributions of reverse-flow time fraction and surface velocity, it is also seen that the forced flow has more coherent large-scale vortices. The trajectories of elemental vortices for both cases clearly displayed the fact that the forcing increased the shear layer growth rate and the turbulent intensity and decreased the reattachment length. For the forced flow, large-scale vortices are more evident and the rolling-up behavior of large-scale vortices is more active in the separation bubble. From the cross-correlation between the surface-pressure at reattachment and the streamwise velocity fluctuations, it is seen that more vortex clusters are compressed within the separation bubble with increasing forcing frequency. As the forcing frequency increases, the convection velocity also increases.

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